Rational Function Graphs

Parent: $f(x) = \frac{a}{x}$ $(a \neq 0)$

Vertical Asymptote $x=0$
Horizontal Asymptote $y=0$

Graph: $f(x) = \frac{a}{x-h} + k$

Vertical Asymptote: $x = h$
Horizontal Asymptote: $y = k$

$f(x) = \frac{3x+2}{x-7}$

VA: $\frac{7}{1} = 7$
HA: $\frac{3}{1} = 3$

$f(x) = \frac{ax+b}{cx+d}$

Vertical Asymptote: $x = \frac{-d}{c}$

Horizontal Asymptote: $y = \frac{a}{c}$

$f(x) = \frac{p(x)}{q(x)} = \frac{a_m x^m + \ldots}{b_n x^n + \ldots}$

Ex. $\frac{3x^3-2x+7}{4x^3+15}$

$m=3$ $n=3$ $a=3$ $b=4$

Step 1: Plot x-intercepts (real zeros of p(x)) - x values that make numerator = 0
Step 2: Draw vertical asymptotes (zeros of q(x)) - x values that make denominator = 0
Step 3: Draw horizontal asymptote if it exists

if $m < n$ $y = 0$ is a HA
if $m = n$ $y = \frac{a}{b}$ is a HA
if $m > n$ there is no HA

Example

$$\boxed{y = \frac{3x^2}{x^2 - 1}}$$

$m=2$
$n=2$
$a=3$
$b=1$

Step 1: Find and plot x-intercepts. (Set numerator equal to 0 and solve for x.)

$\frac{3x^2}{3} = \frac{0}{3}$

$\sqrt{x^2} = \sqrt{0}$ $x=0$

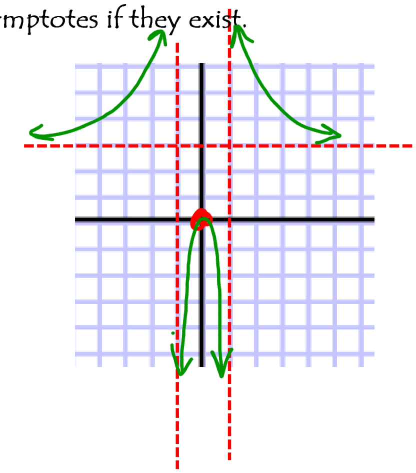
Step 2: Find vertical asymptotes. (Set denominator equal to 0 and solve for x.) Plot dotted vertical lines.

$x^2 - 1 = 0$
$(x-1)(x+1) = 0$
$x = 1, -1$

$x^2 - 1 = 0$
$\sqrt{x^2} = \sqrt{1}$
$x = \pm 1$

Step 3: Find and plot horizontal asymptotes if they exist.

H.A. $= \frac{a}{b} = \frac{3}{1} = 3$



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18. $y = \dfrac{x-4}{x^2-3x}$

$m = 1$
$n = 2$

$m < n$

Step 1: x-intercepts:
$x - 4 = 0$
$x = 4$

Step 2:
$x^2 - 3x = 0$
$x(x-3) = 0$
$x = 0$ $x - 3 = 0$ $x = 3$

Step 3: $m < n$
$1 < 2$
$y = 0$ H.A.